1.

Control chart technique has associated the diagram with statistic test principle and is used to detect data variance caused by assignable cause. Tukey's control chart adopts single observation value test and uses quartiles to set up control limit; the method of the setup of control limit is simple and easy. Alemi (2002) applied Tukey's control chart in medical industry to monitor the medical history of a patient; however, since the data of medical history is very few, it is very difficult to accurately fit population's probability distribution; therefore, Alemi (2002) calculated directly quartiles from the data and set up the control limits for Tukey's control chart. The monitoring performance of control chart can be an objective evaluation only when the probability distribution is known, but since Alemi's (2002) research lacks of the assumption of probability distribution, hence, the monitoring performance of Tukey's control chart can not be evaluated; furthermore, there are no related researches investigating the monitoring performance of Tukey's control chart available.

Average run length (ARL) is suitable to be used as the performance index of control chart of fixed sampling interval and sampling number; in the past, it is used to display the process monitoring capability of control charts such as Shewhart, EWMA and CUSUM, etc.; ARL is, under specific probability distribution assumption, the use of statistical method to display the expected sampling number of control chart in detecting process variance and the occurrence of false alarm, hence, it can express the monitoring capability of control chart better than process mis-judgment probability; therefore, ARL is the frequently used evaluation index in the control chart research.

In manufacturing industry and in continuous manufacturing process or the manufacturing process of chemical solution, each sampling can only extract single observation value, but in this type of process, individual control chart needs to be used to monitor process mean. Tukey's control chart is one type of individual control charts and is suitable to be used for the monitoring of the average mean of single observation value process. Different industry process will have different requirements on the performance of control chart; in many of the industry process monitoring, it will be asked that for control chart in stable process, only one false alarm is allowed for an average sampling frequency of 370.4 times (it means that the ARL for stable process is ARL=370.4); therefore, for many researches of control chart, the width of control limit is decided under this standard; meanwhile, Alemi (2002) had not evaluated the ARL of Tukey's control chart; hence, the control limit width decided by him can not necessarily be applied the process monitoring of the manufacturing industry; therefore, the control limit width of Tukey's control chart must be re-decided according to process monitoring requirement.

In the past research, normal distribution is usually the basic assumption for using control chart to monitor the process; when the observation value seriously violates the normal distribution assumption, the process monitoring performance of control chart will be affected too; Borror et al. (1999), Stoumbos & Reynolds (2000), Calzada & Scariano (2001) once used gamma distribution as asymmetric distribution and t distribution as symmetric distribution to investigate the performance of control chart under non-normal distribution; it is found from the result that most control charts will have obviously worse performance of process monitoring under non-normal assumption than that under normal assumption. Therefore, the control limit width must be decided according to different probability distribution so as to keep the ARL of stable process at 370.4. When Tukey's control chart is used in the monitoring of non-normal process, the control limit width needs to be re-decided so as to let ARL of stable process remain at 370.4, and the capability of detecting drift needs to be evaluated.

The data collection in manufacturing industry is usually easier than that in the medical industry; in the long term manufacturing and production process, massive process observation values can be collected, and accurate fit of the probability distribution and parameter of the population can be done from the massive observation values; in many control chart researches (He et al., 2002; Lin & Chou, 2007) in the investigation of statistical design problem, it is usually assumed that correct population probability distribution has been obtained and it is through probability distribution that the control limit is set up and the monitoring capability index is calculated.

Based on the above motive, Tukey's control chart is used in the process monitoring of the manufacturing industry in this study; Tukey's control chart is set up under the assumption of known population probability distribution and the ARL calculation method is constructed. In this study, it is assumed respectively normal, gamma and t distribution in the population and stable process of ARL=370.4 is used as the basis to decide the best control limit parameter of Tukey's control chart and to test the variance detecting capability of Tukey's control chart; that is, the process monitoring performance under normal assumption and non-normal assumption is evaluated.

2.

When control chart is used in the process monitoring, control chart must be adjusted in the control limit width according to process monitoring need; based on the statistical design view point of control chart, the parameter of control limit will be decided by the allowable process mis-judgment probability; since process status can be divided into process in control (δ=0) and process with variance ($\delta \neq 0$), hence, the parameter k of Tukey's control limits can be decided by type 1 error or type 2 error.

Assume under stable process status, the probability of the occurrence of type 1 error must meet tolerance value α, which can be represented as

$$1-\int_{F^{-1}(0.25)-k\times IQR}^{F^{-1}(0.75)+k\times IQR} f(x)\,dx=\alpha \qquad (8)$$

If the detecting efficiency of process mean drift $\delta\sigma$ is emphasized, then the probability of type 2 error must meet tolerance value β, which can be represented as

$$1-\int_{F^{-1}(0.25)-k\times IQR-\delta\sigma}^{F^{-1}(0.75)+k\times IQR-\delta\sigma} f(x)\,dx=1-\beta \qquad (9)$$

Let k of Eq.(8) or Eq.(9) become decision variable, then Newton-Raphson method can be used to solve the mathematical formula to get the optimum k value. Since Tukey's control chart is one of individual control charts, type 2 error probability can not be reduced through the increase of sample number, therefore, we know that the statistical design of Tukey's control chart can not let type 1 error and type 2 error reach a minimum at the same time; it can be sent that once the mis-judgment probability of one of the processes is reduced, then the mis-judgment probability of another process must be increased, hence, it is only necessary to choose one mathematical formula from either Eq.(8) or Eq.(9) in order to decide k value; if the reduction of misjudgment probability of stable process is to be emphasized, Eq.(8) should be chosen to find k value; if the detecting efficiency of process variance is to be emphasized, then we should choose Eq.(9) to find k value. Many researches will set the type 1 error probability of control chart at 0.0027($ARL(\delta=0)=370.4$) and use it as a common standard value before the variance detecting capability is re-evaluated; if we want to let the type 1 error probability of Tukey's control chart reach 0.0027, then we should use Eq.(8) to decide the k value.

3. An example

In certain chemical raw material process, the concentration of chemical raw material is to be monitored and one concentration is measured in each sampling process. In the manufacturing process of the past one year, 500 observation values of stable process are collected; then Kolmogorov-Smirnov test is done to find out that the observation value obeys normal distribution (P-value = 0.75), μ=33.52, σ=0.423; in the future, if Tukey's control chart is to be used to monitor the process and the mis-judgment probability of the drift of mean 2σ(δ=2) must be smaller than 0.6667(β=0.6667), then the k value of control limit must be decided and the probability value of type 1 error must be evaluated.

Let $f(x)$ is PDF of normal distribution with parameter μ=33.52 and σ=0.423; let p be equal respectively to 0.25 and 0.75 and substitute them into Eq.(5), then we can obtain $F^{-1}(0.25)=33.23469$ and $F^{-1}(0.75)=33.80531$, *IQR*=0.570618; then we substitute $F^{-1}(0.25)$, $F^{-1}(0.25)$ and *IQR* into Eq.(9) and let δ=2, β=0.6667, then *k* is used as the decision variable to solve Eq.(9), we can then get *k*= 1.302. Furthermore, let δ=0 and substitute $F^{-1}(0.25)$, $F^{-1}(0.25)$, *k* and *IQR* into Eq.(7), we can then obtain the probability of type 1 error to be 0.0151(*ARL*(0)=66.39).

4.

In this section, the probability distribution selected in the last section is going to be used in the calculation of the ARL of Tukey's control chart. When the population is gamma distribution of *a*=4, *b*=1, its standard deviation is 2, $F^{-1}(0.25)$ and $F^{-1}(0.75)$ are respectively 2.5353 and 5.1094 and *IQR*=2.5741, then from Eq.(6), it can be calculated that UCL =8.9706 and LCL=-1.3258. Substitute UCL, LCL, δ=0 and σ=2 into Eq.(7); let the *f*(*x*) of Eq.(7) be the PDF of gamma distribution of *a*=4, *b*=1, we can then obtain $P(\delta=0)=0.0217$ of Gam(4,1) with *ARL*(δ=0)=46.14. If we apply the ARL calculation method to other probability distributions, we can then obtain the result of table 1.

It can be seen from table 1 that *ARL*(δ=0) and *ARL*($\delta\neq$0) under all probability distribution assumptions are not equal and the process monitoring capabilities under all probability distribution assumptions thus can not be compared. Next, *k* value will be adjusted in this study to let *ARL*(δ=0) of Tukey's control chart approach 370.4; then compare *ARL*($\delta\neq$0) and investigate the differences in detection capability under all kinds of probability distribution assumptions; then let α=0.0027 in Eq.(8) and substitute the PDF of all kinds of probability distributions into *f*(*x*) of Eq.(8) to find *k* value, then ARL at each drift can be calculated, with the result shown in table 2.

In table 2, *ARL*(δ=0) of Tukey's control chart under all probability distribution assumptions approach 370.4; as we compare the ARL at each drift, it can be seen that when the probability distribution assumption of population gets more away from normal distribution, the efficiency of detecting process variance by Tukey's control chart will become worse; it is especially true that two probability distributions of Gam(1,1) and t(4) will become slower.

Table 2 also shows the ARL of Shewhart's individual control chart. Shewhart's individual control chart uses $\mu \pm k\sigma$ to set up the control limit, under normal distribution assumption, Shewhart's individual control chart of *k*=3 has *ARL*(δ=0)=370.4; meanwhile, we also change *k* of Shewhart's individual control chart to let *ARL*(δ=0) of Shewhart's individual control chart under all kinds of probability distribution assumptions approach 370.4, and finally, the ARL at different drift is calculated. It can be seen from table 2 that Shewhart's individual control chart has the same process monitoring capability than Tukey's control chart.

5.

Tukey's control chart is one type of individual control charts. It adopts quartiles method to set up control limit; since the setup method is very simple and easy to use, it is thus used in the past in the data monitoring in medical industry. In this study, Tukey's control chart is used in the process monitoring of manufacturing industry; it is assumed that the population probability distribution of process can be obtained through massive historical data fit, and the control limit of Tukey's control chart is set up under known probability distribution and the formula for calculating average rub length is developed. After ARL comparison, it is fund that Tukey's control chart has the same process monitoring performance as Shewhart's individual control chart. When process observation value seriously violates normal distribution assumption, Tukey's control chart will become slower in detecting process variance than the normal distribution assumption. Therefore, it is suggested that when Tukey's control chart is used to monitor non-normal process, the observation values should be first converted to data very similar to normal distribution (For example, the method of Box & Cox (1964)) before the control limit is set up and process is monitored so as to enhance the performance of detecting process variance.

When observation value is difficult to be collected and the probability distribution of population is difficult to be accurately fit, then only few observation values need to be used to set up Tukey's control chart, and such monitoring capability is going to very different than the value displayed in this study; it is suggested that future study can focus on this aspect for further investigation.